Universidad de Puerto Rico
Recinto de Río Piedras
Departamento de Economía
Unidad de Investigaciones Económicas

Structural Change and Factor Prices

por
Jaime del Valle

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Prefacio

El trabajo que hoy presentamos del Dr. Jaime del Valle resulta de particular interés por varios motivos. En primer término, ofrece una elegante presentación crítica de la teoría del capital. Para ello adopta la perspectiva neoricardiana, basándose particularmente en las aportaciones de Sraffa y Pasinetti, para replantear la formulación teórica del modelo de insumo-producto. Desde ese punto de referencia, analiza teóricamente cómo cambios en los precios relativos de los factores resultan en promover cambios tecnológicos.

El autor expande la discusión teórica, enmarcada, como se indicó, en este debate de la teoría del capital, utilizando información referente a la economía de Puerto Rico para el período de 1963 a 1977. Ese análisis empírico le permite corroborar que las relaciones capital-producto y capital-trabajo varían con cambios en el valor del capital, aún cuando no ocurran transformaciones en los modos de producción.

Resulta entonces que el trabajo no sólo tiene relevancia teórica para el economista profesional, sino que adquiere particular pertinencia para Puerto Rico hoy. Las propuestas de cambios en las disposiciones de la Sección 936 tienen el efecto de modificar los precios relativos de los factores de producción: reducir los beneficios contributivos concedidos por esta disposición tiene el efecto de aumentar el precio relativo del capital. En ese contexto, el argumento desarrollado por del Valle llevaría a una predicción de cambio estructural en la economía de Puerto Rico. Aunque el autor no trata este tema, es evidente que el tipo de análisis propuesto en este trabajo puede ser de utilidad para efectuar estudios futuros que ayuden a predecir mejor lo que se puede esperar que ocurra en nuestra estructura económica, en la eventualidad de cambios en las disposiciones contributivas federales que inciden sobre las inversiones en la isla.

Ramón J. Cao García, Ph.D.
Director UIE

El Autor

Jaime L. del Valle Caballero es Catedrático Auxiliar en el Departamento de Economía del Recinto de Río Piedras de la Universidad de Puerto Rico, donde labora desde 1989.

El profesor del Valle cursó sus estudio de bachillerato en Economía en el mismo Recinto de Río Piedras de la Universidad de Puerto Rico, y obtuvo los grados de maestría y doctorado en las Universidades de Cambridge y Manchester, en Inglaterra, respectivamente.

Los campos de interés del doctor del Valle son la teoría política (economía scraffiana), el análisis del progreso tecnológico y los modelos dinámicos de crecimiento económico.

STRUCTURAL CHANGE AND FACTOR PRICES

Jaime del Valle

Introduction

> *"The theoretical implications of the foregoing results are rather far reaching with reference to one of the most vexed questions in capital theory: the question of whether* ***-at any given state of technical knowledge-*** *there is any relationship between changes in the rate of profit and changes in the 'quantity of capital' per unit of labour.*
> *It has long since been discovered that when, in an economic system, the rate of profit is changed, but the physical capital goods remain the same (...) the* ***values*** *of these physical capital goods normally change, in a way which may be very different indeed from one commodity to another."* (Pasinetti [1966], pp. 512-513; emphasis added)

With these words Pasinetti summarised the implications of the theoretical developments of capital theory. It is our interest in this paper to put together these theoretical elements, with the preliminary results on the system of relative prices obtained for the Puerto Rican economy.

It should be stated at this early stage that the analysis in this paper will be carried out, as Pasinetti explicitly mentions, for a "given level of technical knowledge", that is in the absence of technical change. This issue will be left to a forthcoming paper. We will argue that traditional input-output analysis considered structural change only as a result of technical progress, being totally unable to consider the possibility of structural change in the absence of technical progress. This last possibility is precisely, as the quote from Pasinetti points out, what the recent contributions in capital theory demonstrated.

In the first part of this paper we will discuss the meaning of the concept of "structural change" implicit, or sometimes explicit, in traditional input-output literature. We will argue that, given the theoretical framework in which such analysis has been set, the scope and implication of that kind of analysis is greatly reduced and possibly even made trivial. At the end of that discussion we will propose an alternative view of the concept "structural change".

The second part of the paper is devoted to the formal analysis of the capital-output and capital-labour ratios, variables which we understand to be at the core of the analysis of structural change, and through which we will integrate the recent theoretical developments of capital theory. We will discuss the formal arguments related to the measurement and interpretation of these variables, and will explain the role that the system of prices plays in this analysis. The analysis will be carried out not only in terms of various rate of profit scenarios, but also in terms of what we may call the ***interindustrial*** and the ***sectoral*** analyses. This is done in order to show that the analytical differences between traditional input-output and the "Sraffa-Pasinetti" fameworks are due not only to the traditional disregard of the advances in capital theory, but also to the ***level*** of the analysis (Steedman [1988]).

On the Meaning of "Structural Change"

Very early in his writing Leontief told us that, from the framework of an explicitly formulated economy, "change" could be analysed either from the point of view of the dynamics of the system, or from that of the changing "structure" of the system. He sees a clear distinction between the process of structural change and the processes going on in a growing economy. With this dichotomy in mind, he defines the "structure" of an economy in the following way:

> "The input-output structure of any particular industry is described by a set of 'technical coefficients', a_{ik}, each of which states the amount of each particular input absorbed by that industry per unit of its own output." (Leontief [1953], p.18)

It clearly follows that:

> "Economic systems with identical sets of input-output coefficients can be said to be ***structurally identical***, and systems with unlike technical matrices ***structurally different***. Structural change, in other words, is a change in the structural matrix of the system." (Ibid, p. 19, emphasis added.)

Seventeen years later, and after many empirical studies of a number of economies from all over the world, the above definition of structural change, and the methodology for its analysis, was expounded by Anne Carter in her Structural Change in the American Economy as follows:

> "Input-output coefficients enumerate the amounts an industry purchased from all other industries and from value added, per unit of output. Thus, each input coefficient shows the requirement for a particular input, per unit of a particular output. A column of coefficients, then, gives a detailed quantitative description of the ***technique of production*** used by a sector, a sort of recipe for its output, with specifically enumerated inputs as ingredients. As an input-output coefficient table includes a column of input-output coefficients for every sector, it gives a comprehensive ***structural description*** of the entire economy for a particular year." (Carter [1970], pp. 7-8; emphasis added)

Given this definition of the structure of an economy, she concludes:

> "In the present study structural change means changes in the input-output coefficients." (Ibid, p.217)

We can see from the above statements that no matter how much the methodology for the empirical analysis of an economy might have changed, the working definition of "structural analysis" has not changed from that originally given by Leontief. Structural analysis is simply the description of the physical magnitudes as expresed by the input-output coefficients. Also, we can note the interchangeable use of the words "structural" and "technical", as if wanting to convey a particular idea of what the structure of an economy is, as no more than a simple description of some measured physical magnitudes.[1] With this identification of the concepts

1. For other examples of this see also Stäglin, Reiner & Hans Wessels "Intertemporal Analysis of Structural Change in the German Economy" in Carter, A. & A. Brody [1972:b], pp. 370-392, and P. N. Mathur "An efficient Path for the Technological Transformation of an Economy". Chapter 3 in Barna, T. [1963], pp. 39-56.

"structural" and "technical" it clearly follows that for the traditional input-output analysis, structural change is limited to the case in which the techniques of production change. But more important is the fact that this treatment of structural change ***precludes*** the possibility of changes ***within*** a given technique. Moreover this 'flexibility' of usage between the two terms enables the writers to present the implications of change in the elements of the input-output matrices in the same way as technical change affects the well behaved neoclassical aggregate production function.[2] These results, in terms of the assumed behaviour of the well-behaved neoclassical aggregate production function, are theoretically and empirically incorrect.

The usual or most general procedure for such structural change analysis is to simulate, on the basis of different input-output matrices, what would have been the requirements of output, intermediate inputs, labour, capital or any other desired analytical magnitude, for the satisfaction of a bill of final demand for a given base year. Let us take a very simple example to see this in more detail. We have from Leontief's basic quantity equations system that:

$$
\begin{aligned}
x^{63} &= (I - A)_{63}^{-1}\, y^{63} \\
x^{77} &= (I - A)_{77}^{-1}\, y^{77}
\end{aligned}
\qquad (1)
$$

Given the input-output coefficient matrices A for 1963 and 1977, for example, to satisfy the bills of final demand **y** in each year, quantities $\mathbf{x}^{63}$ and $\mathbf{x}^{77}$ of total output would have to be produced in each of these years respectively. To analyse the "changing structure" of production between these two years, input-output economists traditionally formulate a system along the following lines:

2. See for example Carter [1970], pp. 10-11, Leontief [1953] pp. 32-34 and R. Grosse p. 186 in Leontief [1953].

$$x_{63}^{77} = (I - A)_{63}^{-1}\, y^{77} \tag{2}$$

Since we are keeping the level of final demand equal to that observed in 1977, but are using the input-output matrix of 1963, we interpret vector $\mathbf{x}_{63}^{77}$ as the vector of total production which it would have been necessary to produce in 1977 had there been no changes in the methods of production since 1963. Comparing that vector with the actual vector $\mathbf{x}^{77}$ we attribute the difference in gross production to the apparent "structural change" of the economy. Following the same basic principle we could formulate the system for the analysis of the change in the labour and capital requirements as follows:[3]

$$\begin{aligned} e_{63}^{77} &= a_{63}(I - A)_{63}^{-1}\, y^{77} \\ s_{63}^{77} &= C_{63}(I - A)_{63}^{-1}\, y^{77} \end{aligned} \tag{3}$$

In these equations e is the (scalar) number representing total labour employed in the system, **a** is the row vector of industrial employment per unit of output, **s** is the column vector of the amount of capital used in each industry of the economy and **C** is Leontief's capital matrix, which contains the same industries as matrix **A**.

Nevertheless, input-output analysis has not stayed still and by drawing on the "technological" connotation of the definition of structural change, it has carried forward the implications of its results in terms of the capital-output and capital-labour ratios (or their analogues, the degrees of capital intensity and of mechanization). In this way, relative changes in employment and physical capital have been compared, and statements about what ***must*** have

3. It should be emphasized here that these are a very general formulation of the models used for the analysis of "structural change". We are well aware that as they are formulated here they stress the role of the change in demand as a factor explaining changes in the other variables. For more details of the specific form of these models see Almon [1966], Barna [1963], Carter [1970], Carter & Brody [1970: a & b], Chenery [1959], Gupta & Steedman [1972] and Leontief [1953 & 1985], among others.

happened to these ratios, with all their implications regarding changes in factor prices, substitution and marginal productivies, have been made.

Note that for the input-output analysis of structural change, carried out along these lines, there has been no need to make use of the value magnitudes of capital, but we cannot compare the capital values implied by two different quantities of physical capital, independently of their prices and distribution, and this analysis is completely missing in the input-output framework. Given that, for unchanged techniques of production, the ***values*** of the (unchanged) physical capital goods may, and most probably will change, we think that the disregard for the value measure of capital invalidates any attempt to apply the concepts of capital intensity and degree of mechanization to traditional input output theory. It is for these reasons that we find the traditional definition of the structure of an economy rather 'trivial', for it fails to give any useful description of the economy in terms of the degrees of capital intensity and mechanization.[4] In fact, such a framework has its validity limited only to an 'accounting' practice of the interrelationships of physical flows and stocks, totally unable to go any further in the description of the economy.

For our analysis we will propose an alternative definition of the structure of the economy. When we talk about structural changes in an economy, we mean the internal changes as they are reflected in the relationship between the level of investment necessary to increase the flow of output by one unit, and by the employment required by each additional unit of investment. These relationships are precisely encompassed by the capital-output and capital-labour ratios. Moreover we will consider the analysis of these magnitudes, in the absence of changes in the technical methods of production. What we want to show is precisely the way in which the actual value of capital changes as nothing but the distribution of income changes.

4. It is only fair to mention that in Carter's book [1970] there is a brief but honest discussion of the problem of measuring capital goods, although the problems raised by her do not follow the same line of reasoning as the well known capital controversy. See Carter [1970], pp. 146-ff.

Frank Englmann, in an article on structural change and heterogeneous capital, has stated that:

> "Structural change implies that the composition of capital goods invested in the past, present and future is different: the capital stock is heterogeneous over time." (Englmann [1987], p. 185.)

As we just said before, we can measure these internal changes by means of capital-output and capital-labour ratios, or as we have termed them above, the degrees of capital intensity and mechanization. What was previously called the "structure" of an economy we will now call, using Sraffa's terminology, the "technical methods of production".[5] Each input-output column represents the actual ***method of production*** used in each industry, and the combination of methods for each production activity we shall call the technique of the system, and we can describe them as in the input-output sense of a recipe of specified inputs for the production of a unit of a particular commodity. That is to say, we are calling the technical methods of production what Leontief, Barna, Carter and others called the structure of the system. Nevertheless, we must state that the fact that we have labelled the actual combination of methods of production represented in the input-output matrix the "technique" of the system does not mean that there is only one known and available technical method of production for each commodity, as in the quote from Carter above. What it means here is that given all the alternative methods of production known for each commodity, the actual system consists of those methods which, given the present distribution of income, are the most profitable.[6] To analyse the changes in the structure of the economy we need more information than a simple new coefficient matrix; we need information about prices and the distribution of income between wages and profits.

5. It is interesting to note that, although Sraffa's work is based on well known input-output relationships, he nowhere talks of the "structure" of the economy to refer to the "technical methods of production".

6. See for example J. Robinson [1956].

The Capital-Output and Capital-Labour Ratios

The capital-output ratio (k/o) denotes the total value of capital used in each industry divided by the value of that industry's total gross output, while the capital-labour ratio (k/l) equals the same value of capital divided by the total amount of labour employed in each industry per unit of its output. On the other hand, the overall capital-output (K/O) and capital-labour (K/L) ratios denote the sum total of the value of capital used in each and every industry divided by the total value of gross output and by total employment respectively.

As we mentioned before, there are two alternative ways in which we could measure these ratios. The first one is to follow the ***interindustrial*** analysis but including the effect of the system of prices on the measurement of the value of capital. The second approach, which we is followed here, is based on the ***sectoral***, or vertically integrated approach.

In terms of the ***interindustrial*** approach we measure the value of capital from the input-output matrix **A**. Similarly, the labour inputs used in the capital-labour measures are the direct labour requirements vector **a**.

The ***sectoral*** analysis, on the other hand, is based on the vertically integrated notions of ***direct and indirect*** units of capital (**H**) and labour (**v**) required in the whole system, instead of the direct magnitudes **A** and **a**.

We can write the ***interindustrial*** capital-output and capital-labour ratios in the following way:

$$\left(\frac{k}{o}\right)_i = \frac{pAe_i}{pe_i} \qquad \left(\frac{K}{O}\right) = \left(\frac{pAx}{px}\right) \tag{4}$$

where $\mathbf{e}_i$ is an ith unit vector with 1 in its ith coordinate, conformable for multiplication.

$$\left(\frac{k}{l}\right)_i = \frac{pAe_i}{ae_i} \qquad \left(\frac{K}{L}\right) = \left(\frac{pAx}{ax}\right) \tag{5}$$

On the other hand for the ***sectoral*** ratios we can write, following Varri & Marzi [1977] and Pasinetti [1981]:

$$\left(\frac{k}{o}\right)_i^* = \frac{pHe_i}{pe_i} \qquad \left(\frac{K}{O}\right)^* = \frac{pHy}{py} \tag{6}$$

$$\left(\frac{k}{l}\right)_i^* = \frac{pHe_i}{ve_i} \qquad \left(\frac{K}{L}\right)^* = \frac{pHy}{vy} \tag{7}$$

Note that although we have made use of Pasinetti's vertically integrated matrix **H**, we did not write the formulas for the capital-output and capital-labour[7] ratios in terms of his k "units" of productive capacities because, for empirical purposes, there is no possible transformation between the observed input-output magnitudes and the vertically integrated "units".

Before going any further we should make it clear that, although we have presented these two ways of measuring the capital-output and capital-labour ratios in terms of the interindustrial and the sectoral measures, there is a simple transformation from one to the other. Bearing in mind that matrix **A** is the matrix of direct requirements of commodity i per unit of ***gross*** output of commodity j, while matrix **H** is the matrix of direct and indirect

7. Ochoa refers to this measure of sectoral capital-labour ratio as the "vertically integrated organic composition of capital", a very ingeneous marxist interpretation of a simple concept.

requirements of commodity i for the production of a unit of commodity j as ***net*** output, we can write:

$$pAx = pA(I - A)^{-1}y = pHy$$
$$ax = a(I - A)^{-1}y = vy \qquad (8)$$

From equations (4) and (6), we know that **pA** is the interindustrial value of capital, while **pH** is the sectoral value of capital, while on the other hand vectors **a** and **v** are the direct requirements of labour per unit of ***gross*** output and the direct and indirect requirements of labour per unit of ***net*** output respectively; therefore the equations in (8) can be interpreted as showing that the vertically integrated total value of capital and labour requirements needed for the production of net output will always be equal to the interindustrial total value of capital and labour needed for the production of gross output respectively.

Another thing that we notice from the above system of equations is the introduction of the system of prices in our analysis of structural change, a magnitude which was missing in the traditional input-output analysis.[8]

Since we want to emphasize the analytical difference between the interindustrial and the sectoral magnitudes, from now on we will only consider the individual industries' measures. Moreover, it is also claimed that it is at this level that these magnitudes show their "purely

8. In the above mentioned work of R. Grosse he states that careful analysis has been given of the system of relative prices. This has nothing to do with the role that relative prices assume in our model. For him prices determine the technical possibility of substitution, while in our model prices influence the movement of the value magnitudes of capital goods, irrespective of any considerations of substitution. (See on this issue Pasinetti [1977].) Anne Carter makes a similar analysis to that of R. Grosse of the role of prices in her work. (Carter, loc. cit. pp. 156-157)

technical meaning" (Pasinetti [1981], p. 214) and also it is the level at which these magnitudes originate their changes (Steedman [1983]).

Substituting the expression for the system of prices in equations (4) to (7) we have:

Interindustrial:

$$\left(\frac{k}{o}\right)_i = \frac{\left[v(I - rH)^{-1}\right] A e_i}{\left[v(I - rH)^{-1}\right] e_i} \tag{9}$$

$$\left(\frac{k}{l}\right)_i = \frac{\left[v(I - rH)^{-1}\ w'\right] A e_i}{a\ e_i} \tag{10}$$

Sectoral:

$$\left(\frac{k}{o}\right)_i^* = \frac{\left[v(I - rH)^{-1}\right] H e_i}{\left[v(I - rH)^{-1}\right] e_i} \tag{11}$$

$$\left(\frac{k}{l}\right)_i^* = \frac{\left[v(I - rH)^{-1}\ w'\right] H e_i}{v\ e_i} \tag{12}$$

Note that the capital-output ratios equations (9) and (11) are not dependent upon the choice of numéraire. On the other hand the capital-labour ratios (10) & (12) are indeed affected by the choice of the numéraire-commodity. Nevertheless the uniform wage rate only "scales" the previously computed value of capital from the capital-output ratios, without affecting the behaviour of the system of prices. Whenever we compute these ratios we will follow the convention of expressing prices in terms of the standard commodity.

These formulations might seem quite complicated at first, but on closer inspection we can see that they enable us to reflect in greater detail the relationships between the technical methods of production -as reflected in the technical coefficients **a** and $\mathbf{A}=[a_{ij}]$, and thus **v** and **H**-, the distributive variables w and r, and with these the system of prices.

Since our actual system of prices shows a continuous change in prices as the rate of profit is varied, we must expect from the above system of equations (4) to (7) that our measured quantities of capital in the system change too, without necessarily having to wait for changes in the methods of production. In other words, given our initial institutional assumption that the rate of profit is determined from outside this system of equations, we will observe that, in general, $\frac{d}{dr}\left(\frac{k}{o}\right) \neq 0$, even if the technical methods of production do not change.

From equations (9) and (11), for example, we can see that the capital-output ratios can be expressed as a function of the ratio between the value of the different capital goods used in sector "i" and the price of commodity "i". The potential movement of the capital-output and the capital-labour ratio is given by the result of the "inner product" between **v** and $(\mathbf{I}-r\mathbf{H})^{-1}$ and matrices **A** or **H**, or more simply $\mathbf{p}(r)\mathbf{A}$ or $\mathbf{p}(r)\mathbf{H}$.

- Now **v** shows the units of vertically integrated labour, which we can also call "embodied" labour, while we can denote **vH** as the labour embodied in the capital goods directly and indirectly required for the production of a unit of commodity i as net output, that is "dead labour". We know, again, from the Perron Frobenius theorems, that all the elements of matrix $(\mathbf{I} - r\mathbf{H})^{-1}$ are increasing functions of the rate of profit r, so that as r increases the quantities of "dead" labour increase their importance in determining prices, given the increased amount of profit which accrues to them. That is, the "weight" carried by dead labour in the determination of prices increases as r increases. But, on the other hand, and by the same principle, the importance of embodied labour diminishes continuously. Finally, since the rate of increase of the different components of $(\mathbf{I} - r\mathbf{H})^{-1}$ is not the same, the combined effect of the different components of the price vectors may move in alternating directions. Depending upon the relative magnitudes and rates of change of the different elements of that "inner product", prices will increase or decrease or alternate, as the rate of profit changes, and with these our measures of the capital-output and the capital-labour ratios. As an example we have reproduced in Table 6.2.1 the result of the inner product of $\mathbf{v}$ and $(\mathbf{I} - r\mathbf{H})^{-1}$ w^{*}[9] for the case of price 29 in 1963 under the uniform rate of profit scenario. Each element in any given column stands for the product of the individual elements of vector $\mathbf{v}$ by the corresponding row element of column 29 of $(\mathbf{I} - r\mathbf{H})^{-1}$, for the corresponding r/R. In the last row we print the column sum, i. e. the price of the commodity at the different values of r.

As we can see from that Table, the first column contains only the units of direct and indirect requirements of labour -$\mathbf{v}$- (times the wage rate), a magnitude which decreases in importance continuously as the rate of profit increases. At the same time, we find that as soon as the rate of profit is positive most other components of that price begin to increase their importance in the determination of prices. Nevertheless, that initial participation of the other commodities is more than counterbalanced by the decrease in the embodied labour component, so that the price decreases. At rates of profit obtained for values of $0.15 < r/R < 0.55$, the continuous increase in most of the "dead labour" components reverses the trend in price, making it increase, but the appearance of other decreasing "dead labour" components finally overturns this trend making the price of commodity 29 decrease again.

These equations also show that apart from the particular movement of the elements of the "inner product" of $\mathbf{v}(\mathbf{I} - r\mathbf{H})^{-1}$, these effects are further weighted by the relative importance of the different elements of each column of matrices **A** and **H**. That is, these prices multiply the different entries in the rows of matrices **A** or **H** (depending on the level of analysis) so that, given differences in the technical methods of production, the overall effect of the change in the rate of profit on the measure of the capital-output and capital-labour ratios will be weighted

9. Since the standard wage enters the product as a scalar multiple, thus leaving unaffected the basic results we want to show, we have multiplied $\mathbf{v}\,(\mathbf{I} - r\mathbf{H})^{-1}$ by the standard wage so that the column sum adds up to the price of commodity 29.

by the importance of the different commodities as inputs in the production of each commodity and itself. This also implies that the possibility of capital reversing depends not only on the behaviour of prices, but on the weight placed by the elements of matrices **A** and **H**. Consider, for example, an industry whose production requires as inputs commodities with "highly" fluctuating prices (say, for example, commodities whose prices are non-monotonic, all in the same direction, and with a relatively high coefficient of variation), while itself having a relatively "linear-near-constant" price with respect to the rate of profit; we should expect that its capital-output and capital-labour ratios show the same trend as that of the prices it uses as inputs.

We can formalise these arguments about the movements of the capital-output ratio in the following way:[10]

$$\frac{d}{dr}\left(\frac{k}{o}\right)_i^* \gtreqless 0 \text{ according as } \left(\frac{\dot{p}H_i}{pH_i}\right) \gtreqless \left(\frac{\dot{p}_i}{p_i}\right) \tag{13}$$

We could call the first term in the above expression the "overall value-of-capital effect", which reflects the effects, on the value of capital in the jth industry, of the changes in its inputs' prices, as the rate of profit changes, and call the second term the "own price effect", which reflects the effect in that same industry of the change in the price of its own good, as the rate of profit changes. We could say that these two effects are the analogues of Pasinetti's "capital intensity" and "price effect", that he used to explain the behaviour of the different prices. The main difference here is that our "overall value-of-capital effect" depends on ***all*** prices, while Pasinetti's capital intensity is a relative measure between two prices.

10. For notational convenience we are using the dot over the variable to represent the derivative with respect to r. Also, to avoid duplication, and since the basic expression is similar exept for the **H** matrix, we shall write only the condition for the sectoral analysis.

TABLE 6.2.1

The 43 elements of $v(I - rH)^{-1} w^s$ for Sector 29 (Electric and Irrigation Services), 1963 for values of r/R from 0 to 0.95

r/R	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
1	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0010	0.0012	0.0014	0.0016	0.0019	0.0022
2	0.0000	0.0001	0.0001	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0010	0.0012	0.0015	0.0017	0.0020	0.0023	0.0026	0.0029	0.0033	0.0037
3	0.0000	0.0006	0.0013	0.0019	0.0026	0.0032	0.0038	0.0044	0.0050	0.0055	0.0059	0.0064	0.0067	0.0070	0.0072	0.0073	0.0072	0.0071	0.0068	0.0063
4	0.0000	0.0001	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017	0.0017	0.0018
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
7	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011	0.0013	0.0015	0.0017
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
10	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008	0.0009	0.0011	0.0012	0.0014	0.0016	0.0019	0.0021	0.0024	0.0027	0.0031
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0001	0.0002	0.0003	0.0005	0.0007	0.0008	0.0011	0.0013	0.0016	0.0019	0.0022	0.0026	0.0031	0.0036	0.0042	0.0050	0.0058	0.0069	0.0081
13	0.0000	0.0002	0.0003	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011	0.0011	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0009
14	0.0000	0.0001	0.0002	0.0003	0.0004	0.0006	0.0008	0.0010	0.0013	0.0016	0.0019	0.0023	0.0028	0.0034	0.0041	0.0049	0.0058	0.0070	0.0084	0.0102
15	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0005	0.0006	0.0006	0.0007	0.0008	0.0008	0.0009	0.0009	0.0010
16	0.0000	0.0001	0.0002	0.0003	0.0005	0.0006	0.0007	0.0009	0.0010	0.0011	0.0013	0.0014	0.0016	0.0018	0.0019	0.0021	0.0023	0.0024	0.0026	0.0028
17	0.0000	0.0008	0.0015	0.0022	0.0028	0.0033	0.0038	0.0042	0.0046	0.0048	0.0051	0.0052	0.0053	0.0054	0.0053	0.0052	0.0050	0.0047	0.0043	0.0038
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003
19	0.0000	0.0001	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0008	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
20	0.0000	0.0009	0.0018	0.0026	0.0035	0.0043	0.0051	0.0059	0.0067	0.0074	0.0081	0.0087	0.0093	0.0099	0.0103	0.0107	0.0109	0.0110	0.0108	0.0103
21	0.0000	0.0002	0.0004	0.0007	0.0009	0.0011	0.0013	0.0014	0.0016	0.0018	0.0019	0.0021	0.0022	0.0023	0.0024	0.0024	0.0025	0.0025	0.0025	0.0024
22	0.0000	0.0001	0.0002	0.0004	0.0005	0.0007	0.0008	0.0009	0.0011	0.0012	0.0014	0.0015	0.0017	0.0018	0.0019	0.0021	0.0022	0.0022	0.0023	0.0023
23	0.0000	0.0003	0.0005	0.0007	0.0010	0.0012	0.0014	0.0016	0.0017	0.0019	0.0020	0.0021	0.0022	0.0023	0.0023	0.0023	0.0023	0.0023	0.0022	0.0020
24	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
26	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0005	0.0006	0.0007	0.0007	0.0008	0.0009	0.0010	0.0010	0.0011	0.0011	0.0012	0.0012
27	0.0000	0.0002	0.0004	0.0007	0.0009	0.0012	0.0015	0.0018	0.0021	0.0024	0.0027	0.0031	0.0034	0.0037	0.0041	0.0044	0.0047	0.0050	0.0052	0.0054
28	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009	0.0009	0.0010	0.0011	0.0011
29	0.1015	0.0965	0.0915	0.0865	0.0815	0.0765	0.0715	0.0665	0.0615	0.0566	0.0516	0.0466	0.0416	0.0367	0.0317	0.0267	0.0217	0.0167	0.0116	0.0066
30	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
31	0.0000	0.0002	0.0004	0.0006	0.0008	0.0010	0.0012	0.0015	0.0017	0.0020	0.0023	0.0026	0.0029	0.0031	0.0034	0.0037	0.0040	0.0042	0.0044	0.0045
32	0.0000	0.0001	0.0001	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007
33	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009	0.0011	0.0012	0.0014	0.0015	0.0017
34	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0006	0.0007	0.0007	0.0008	0.0008	0.0008	0.0009
35	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003
36	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002
37	0.0000	0.0001	0.0002	0.0002	0.0003	0.0004	0.0006	0.0007	0.0008	0.0009	0.0011	0.0012	0.0014	0.0016	0.0017	0.0019	0.0021	0.0022	0.0024	0.0025
38	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0007	0.0008
39	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	0.0006	0.0006
40	0.0000	0.0003	0.0006	0.0010	0.0013	0.0017	0.0021	0.0025	0.0030	0.0034	0.0038	0.0043	0.0047	0.0051	0.0055	0.0058	0.0061	0.0064	0.0065	0.0065
41	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
42	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
43	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002
	0.1015	0.1014	0.1013	0.1013	0.1013	0.1014	0.1015	0.1016	0.1017	0.1019	0.1020	0.1021	0.1021	0.1021	0.1020	0.1018	0.1013	0.1007	0.0996	0.0981

With respect to the capital-labour ratios we have that:

$$\frac{d}{dr}\left(\frac{k}{l}\right)_i^{*} = \frac{\dot{p}H_i}{v \cdot e_i} \tag{14}$$

which again will be greater, equal or smaller than zero depending only on whether $\dot{\mathbf{p}}\mathbf{H}_i \gtreqless 0$. From equation (14) we would get the sign of the first component of equation (13) and then, from the price vectors, we take the sign of the second component of (13). Given the sign of the overall expression (13), we may then see the relative importance of each component in the movement of the capital-output ratio as the rate of profit changes.

It is with these formulations of the capital-output and capital-labour ratios that we can soundly base our proposed analysis of structural change, separated from the analysis of technical progress, where the structure of the economy is defined not in terms of its physical or observed composition, as in the case of input-output analysis, but in terms of its "internal" composition of capital and labour. Moreover, the values of capital goods have been reckoned in a consistent way following the recent developments in capital theory, while extending the principle of vertical integration to the units of capital and labour. At the same time our measures of sectoral capital-output and capital-labour are the "system" capital-output and capital-labour, even when we measure individual industries' ratios.[11]

Analysis of the computed capital-output and capital-labour ratios

With the above set of equations we computed the sectoral capital-output and capital-labour ratios for the Puerto Rican economy using three scenarios for the interindustrial and the sectoral analyses. That is, we computed the value of the capital goods, assuming first a uniform profit rate and then using as profit rate denominators sectors 16 and sector 31 respectively, using **A** and **H** as the capital coefficient matrices.

11. We are using the term "system" in the Gupta & Steedman sense, which relates very closely to Pasinetti's process of vertical integration.

We know that if we assume uniformity of the rate of profit, matrix **H** is equal to **A(I - A)**$^{-1}$. When, on the other hand, we assumed differential proft rates, we "scaled" matrix **A** by a diagonal matrix r_j^* with $(1+r_i)/(1+r_j)$ in the main diagonal, and called this scaled matrix $\mathbf{A}^*$. With this matrix $\mathbf{A}^*$ we computed the eigenvalues and the maximum rates of profit, and from it we obtained the price vectors. In effect, under the differential profit rate scenarios, our "capital" matrix becomes $\mathbf{A}^*$, rather than **A**. Since by definition matrix **H** is a derived matrix, the fact that we used this matrix $\mathbf{A}^*$ means that matrix **H** now has a new form. Given that under the differential profit rate we wrote:

$$A^* = \left(\frac{1}{1+r_j}\right) A(I+f) \tag{15}$$

$$p = a \cdot w^s + pA^* + pA^* r_j$$

then matrix **H** becomes:

$$H^* = A^*(I - A^*)^{-1} \tag{16}$$

$$p = v^*(I - r_j H^*)^{-1} w^s$$

instead of the original **A(I-A)**$^{-1}$.[12] The two price systems that will be obtained from (15) and (16) are the same, so the difference in the computation of the value of capital in our structural analysis between the interindustrial and the sectoral models will depend only on the particular form of the $\mathbf{A}^*$ matrix -and thus on the newly derived **H** matrix- and not from any divergence of the system of prices.

12. Note that we have written $\mathbf{v}^*$ instead of **v** because now: $\mathbf{v}^* = \mathbf{a}(\mathbf{I} - \mathbf{A}^*)^{-1}$.

The results of our computations for the capital-output ratios are shown in the graphs in the following pages (Figures 6.2.1 to 6.2.4). The number that appears on the graphs is the eigenvalues of the **A** and **H** matrices (or $\mathbf{A}^*$ and $\mathbf{H}^*$) respectively to which the various capital-output ratios converge.

First of all we should draw attention to the difference between the interindustrial and the sectoral capital-output and capital-labour ratios, not only in terms of their absolute value, but also, more interestingly, in terms of their behaviour. Since the sectoral measures refer to the direct and indirect capital requirements, and since within this web of interrelationships the behaviour of the value of the capital goods indirectly required may be very different from that of the capital goods directly required, it was to be expected that these capital-output and capital-labour ratios would differ both in magnitude and in behaviour.[13] This is corroborated in our results.

Perhaps the most striking features of our results are the marked differences in the behaviour of the capital-output ratios between the different rate of profit scenarios and their dominantly ***monotonic*** nature. In this last respect, we must say that of the 1032 capital-output ratios computed, 896 (87%) were monotonic, while the other 136 (13%) were non-monotonic. Of these non-monotonic capital-output ratios, only two were found with two price-direction-reversals.[14] In general terms we can summarise the results of the monotonic nature of the capital-output ratio by stating that of the 516 interindustrial capital-output ratios 459 (89%) were monotonic while that number for the sectoral analysis was 437 (85%), confirming again our view in the preceding paragraph, but referring now to the monotonicity of the measures. Moreover, in terms of the difference between the rate of profit scenarios, the number of monotonic capital-output ratios is increased in the differential rate of profit scenario with respect to the uniform rate of profit scenario, in both the interindustrial and the sectoral

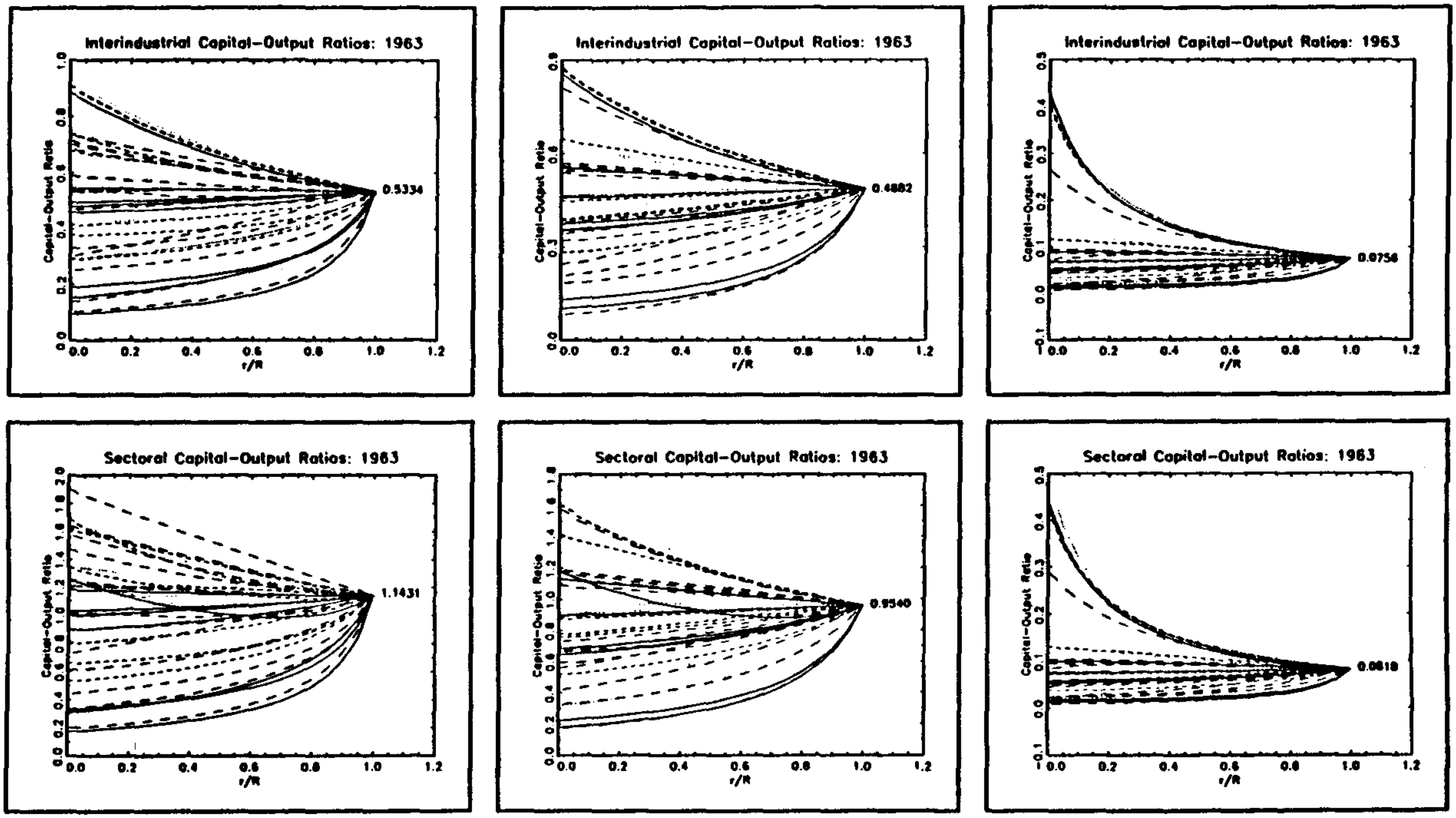
13. See Steedman [1988] for an interesting analysis of the theoretical implications, and analytical differences between the interindustrial and sectoral analyses.

14. Interindustrial k/o, industry #18 (Leather and Leather Products), 1963 for the uniform rate of profit scenario (Table 6.2.2 in Appendix) and sectoral k/o, industry #6 (Bakery Products), 1972 differential profit rate with sector 31 as profit rate denominator (Table 6.2.22 in Appendix).

FIGURES 6.2.1

Capital-Output Ratios 1963, Interindustrial and Sectoral Analyses: Three Scenarios

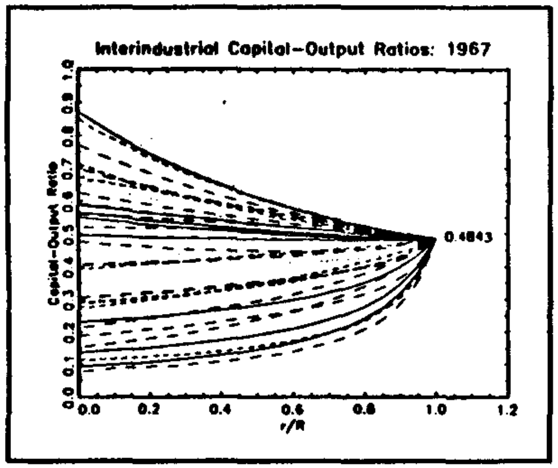
Uniform Rate of Profit	Sector 16 used as Profit Rate Denominator	Sector 31 used as Profit Rate Denominator

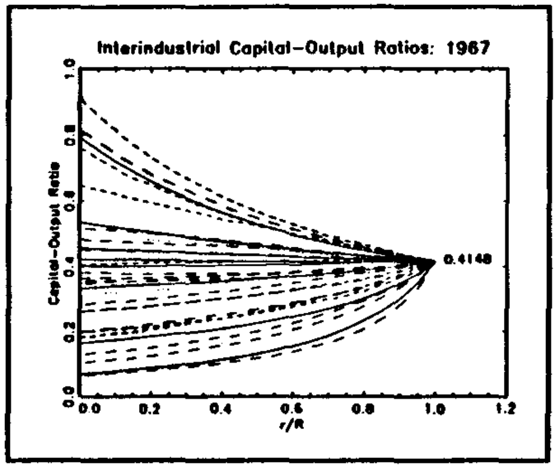


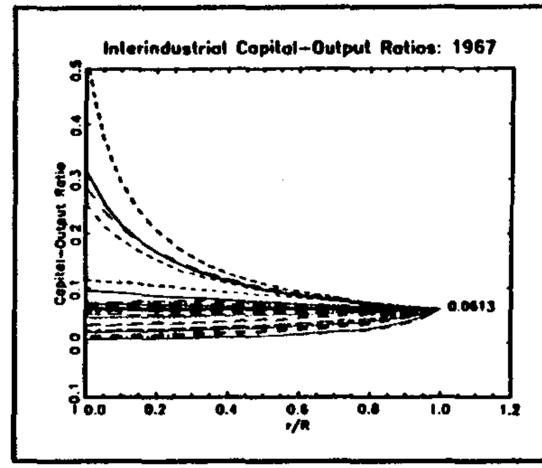
FIGURES 6.2.2

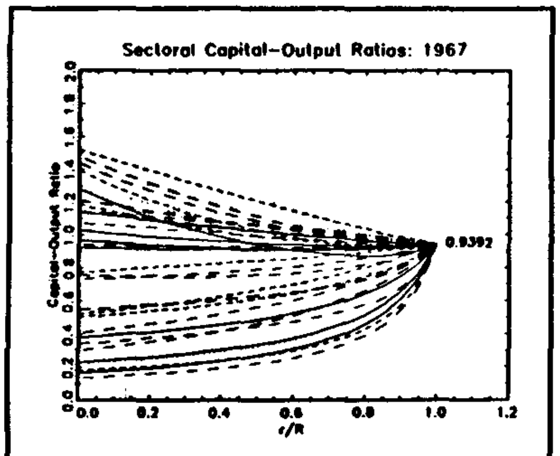
Capital-Output Ratios 1967, Interindustrial and Sectoral Analyses: Three Scenarios

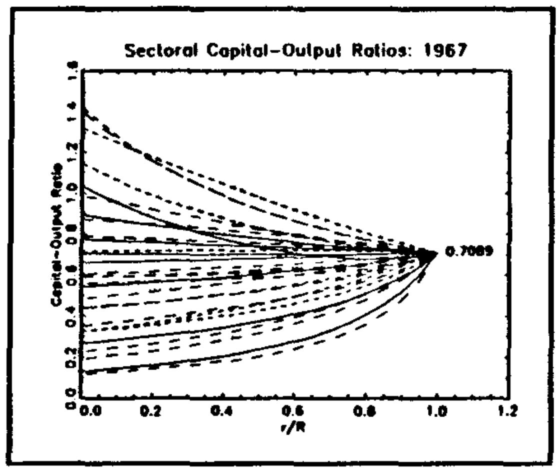
Uniform Rate of Profit	Sector 16 used as Profit Rate Denominator	Sector 31 used as Profit Rate Denominator

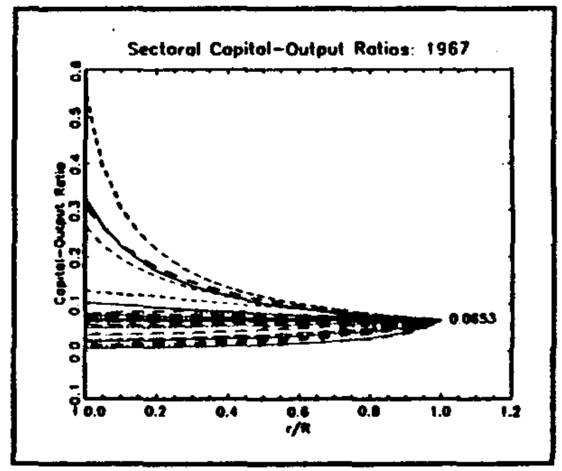








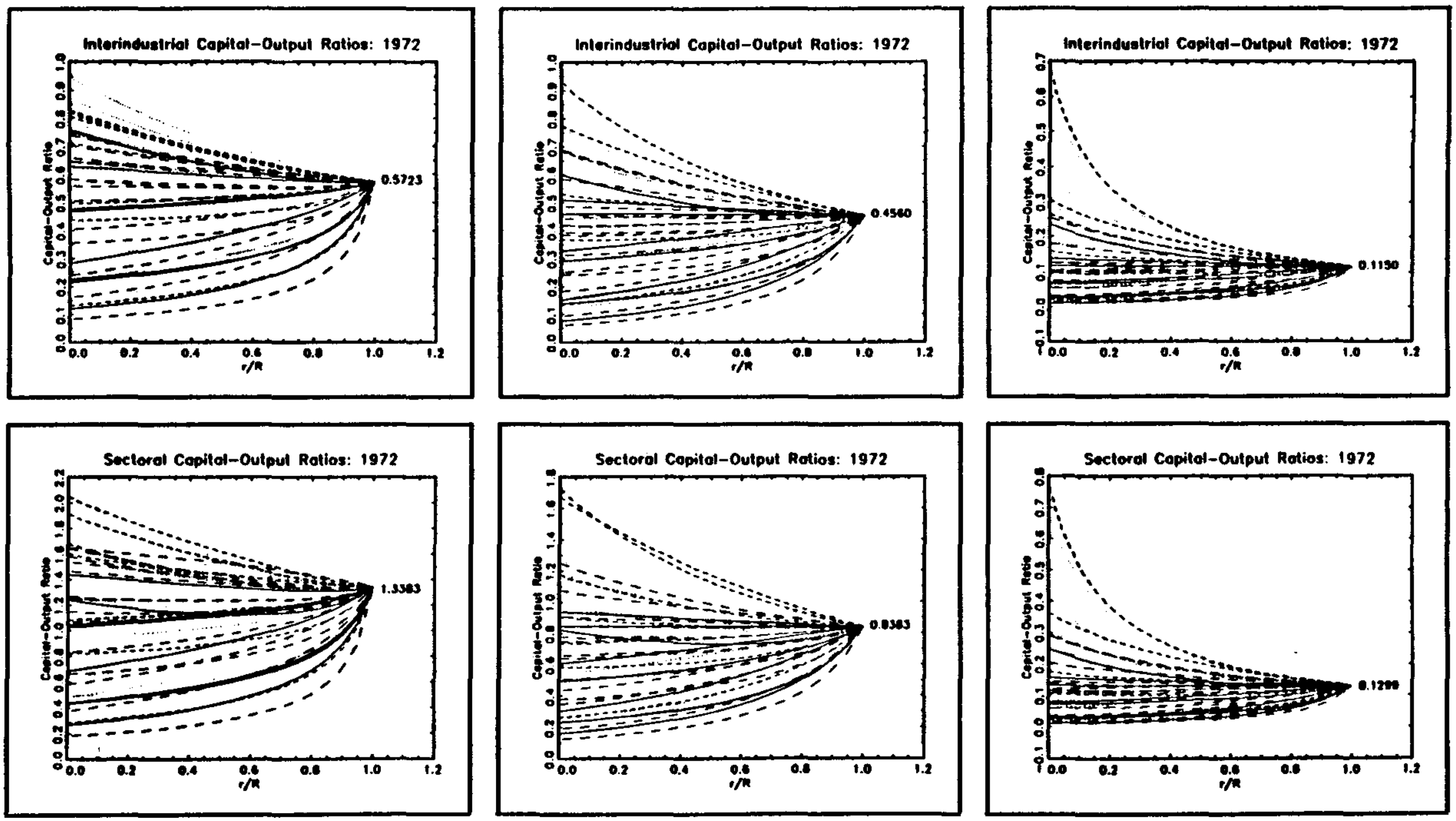




FIGURES 6.2.3

Capital-Output Ratios 1972, Interindustrial and Sectoral Analyses: Three Scenarios

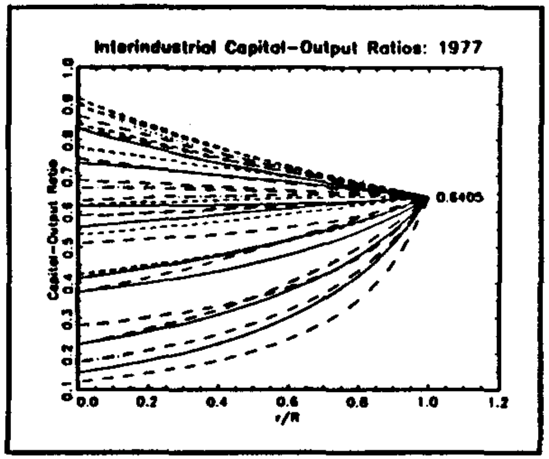
Uniform Rate of Profit	Sector 16 used as Profit Rate Denominator	Sector 31 used as Profit Rate Denominator

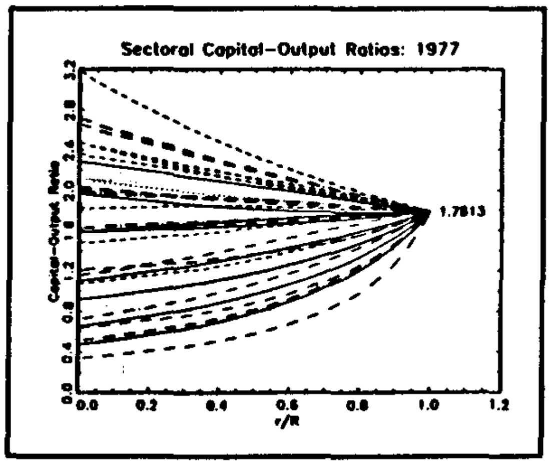


FIGURES 6.2.4

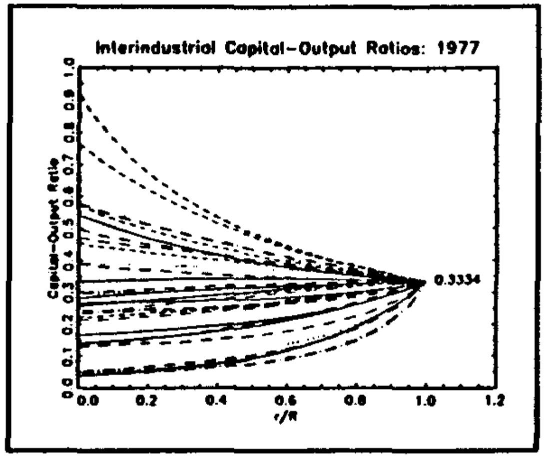
Capital-Output Ratios 1977, Interindustrial and Sectoral Analyses: Three Scenarios

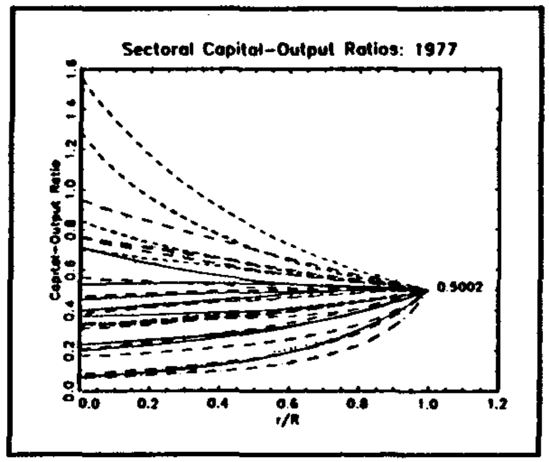
Uniform Rate of Profit



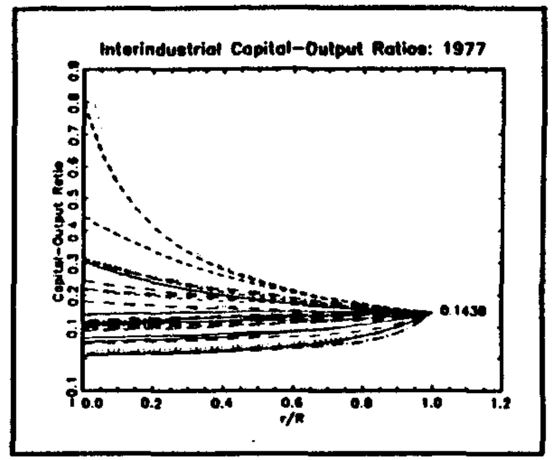


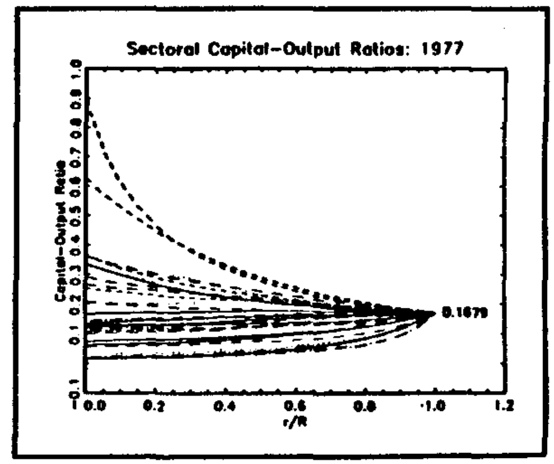
Sector 16 used as Profit Rate Denominator





Sector 31 used as Profit Rate Denominator





analyses.[15] One clearly observable result is that the assumption of differential profit rates has decreased the range of variation of the different industries' capital-output measures. This implies that differential profit rates tend to make different technical methods of production more "structurally similar".[16]

Secondly, we should note that, when the rate of profit equals zero, prices are equal to the direct and indirect units of labour -**v**- so that, in the sectoral analysis we get:

$$\left(\frac{k}{o}\right)_i^{*} = \frac{pH_i}{p_i} = \frac{pH_i}{v_i} = \left(\frac{k}{l}\right)_i^{*} = \frac{vH_i}{v_i}$$

. This last expression is nothing else but the ratio of the direct and indirect units of labour embodied in the capital goods needed for the production of one unit of commodity j as a final commodity over the direct and indirect units of labour required for the production of the same unit of commodity j as a final commodity, i.e. dead labour over embodied labour.

In the interindustrial analysis this equality between the capital-output and the capital-labour does not hold, but the resulting capital-labour ratio becomes:

$$\left(\frac{k}{l}\right) = \frac{vA}{a} = \frac{a(I-A)^{-1}A}{a} = \frac{aG}{a}$$

15. In the interindustrial analysis the number of monotonic capital-output ratios for the uniform rate of profit scenario was 152 (88%), while for the differential profit rate scenarios these figures were 158 (92%) and 149 (87%) for sectors 16 and 31 used as profit rate denominators respectively. In the sectoral analysis the respective numbers are 135 (78%) for the uniform rate of profit scenario and 155 (90%) and 147 (85%) for the differential profit rate scenarios respectively.

16. It is noteworthy that although in the differential rate of profit scenario with sector 31 as denominator, the range of variation of the capital-output ratios is much smaller than when sector 16 is used as denominator, the number of monotonic capital-output ratios is larger in the latter than in the former.

Given the economic interpretation of matrix **G**, the numerator of the last expression can be interpreted as the direct units of labour required as flows in the economy to produce one unit of commodity j as stock, i.e. to produce a unit of the capital good. In this way although there is no parallelism between the equality of the capital-output with the capital-labour ratio in the sectoral analysis with the interindustrial analysis, we can see one in the interpretation of the capital-labour ratios at r=0.

Another thing that we may notice is that the values to which the capital-output ratios "converge" for each and every industry, are the eigenvalues of matrices **A** and **H** respectively. (See Figures 6.2.1 to 6.2.4 for the interindustrial and sectoral cases). From the first theorem of Perron-Frobenius, if λ is the maximum eigenvalue of a matrix **Q** and $\mathbf{x}(\lambda)$ is its associated non-negative eigenvector, then $\mathbf{Q}\ \mathbf{x}(\lambda) = \lambda\ \mathbf{x}(\lambda)$. In our particular case λ^A and λ^H are the maximum eigenvalues of matrices **A** and **H** respectively, while **p**(R) will be the eigenvector associated to them. Furthermore, note that $\lambda^A = 1/(1+R)$ while $\lambda^H = 1/R$, so that $\lambda^H = \frac{\lambda^A}{1-\lambda^A}$. That is, the obtained value of R will be the same for the two levels of analysis (hence the equivalence of our price system expressed either in terms of **A** or **H**). By the above theorem we can write for r=R:

$$p(R)\cdot A = p(R)\cdot\lambda^A_{\max} \qquad p(R)\cdot H = p(R)\cdot\lambda^H_{\max}$$

$$\frac{p(R)\cdot A_j}{p(R)} = \lambda^A_{\max} \qquad \frac{p(R)\cdot H_j}{p(R)} = \lambda^H_{\max} \qquad \forall j \tag{17}$$

As we can see, the use of the sectoral analysis would result in the convergence of the capital-output ratios to the reciprocal of Sraffa's "standard ratio" of ***net*** output to means of production (R), while we must interpret the converging value of the interindustrial analysis as that referring to the ratio of ***gross*** output to means of production. Analogous results are obtained when we use differential rates of profit, with the difference that the eigenvalues (and the values of R) are those of matrices $\mathbf{A}^*$ and $\mathbf{H}^*$.

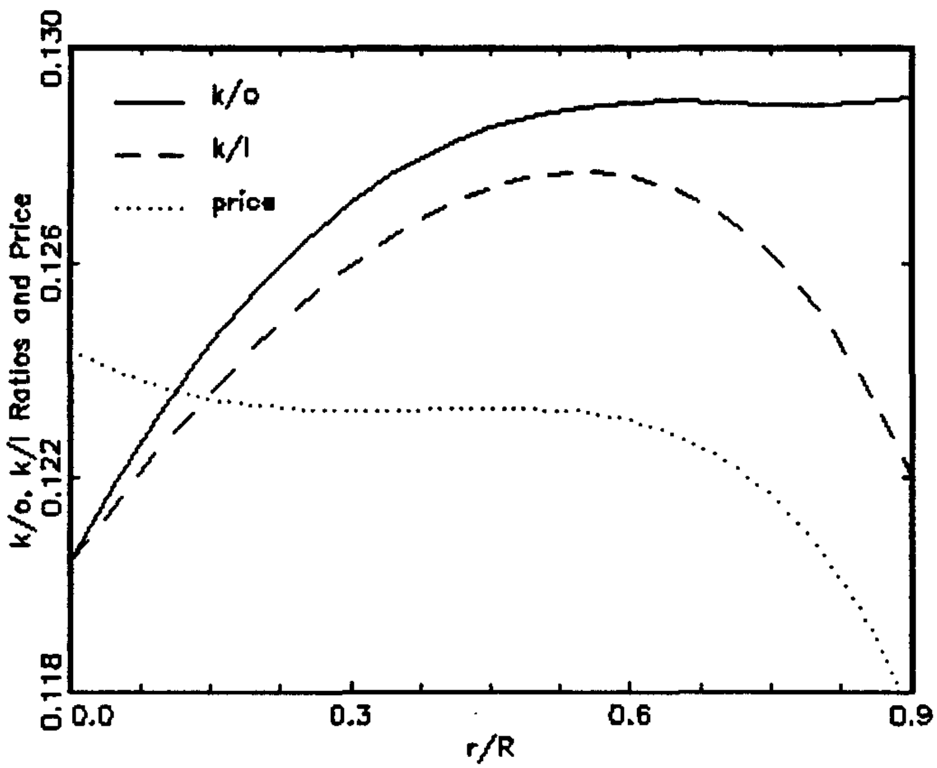
To understand the way in which our analysis throws light upon the importance of the movement of relative prices for the capital-output and capital-labour magnitudes, let us take some examples from the different scenarios.

Let us begin with the sectoral capital-output ratio of industry 6 (Bakery Products) in 1972 for the differential profit rate scenario with sector 31 as profit rate denominator. As we know this industry had a non-monotonic capital-output ratio behaviour, with two direction reversals, while the respective capital-labour ratio was also non-monotonic but with only one direction reversal.

Looking back at the price vector of this industry for this year and scenario, we first observe that this price vector was also the only price vector which we found to have two price direction reversals within the differential profit rate scenarios. Puting together this information, as suggested in equation (13), we can see from Figure 6.2.5 below that when, at low ranges of the rate of profit, the capital-labour ratio was increasing, while the "own price" was decreasing, the capital-output was increasing, meaning that both the "overall value of capital effect" and the "own price effect" were 'pushing' up the capital-output ratio. This was still true during the first direction-reversal of the price vector, and during the 'early' stages of the second direction-reversal of the price vector, at intermediate ranges of the rate of profit. When the first direction-reversal of the capital-labour ratio is observed we then see the first change in the direction of the capital-output ratio, thus making the "overall value of capital effect" the dominant component of the behaviour of the capital-output ratio, throughout this range of the rate of profit. Only at the higher-end of the rate of profit range, when the capital-labour and the own price were decreasing, did the capital-output change direction for the second time, then making the "price effect" the dominant force behind the movement of the capital-output ratio.

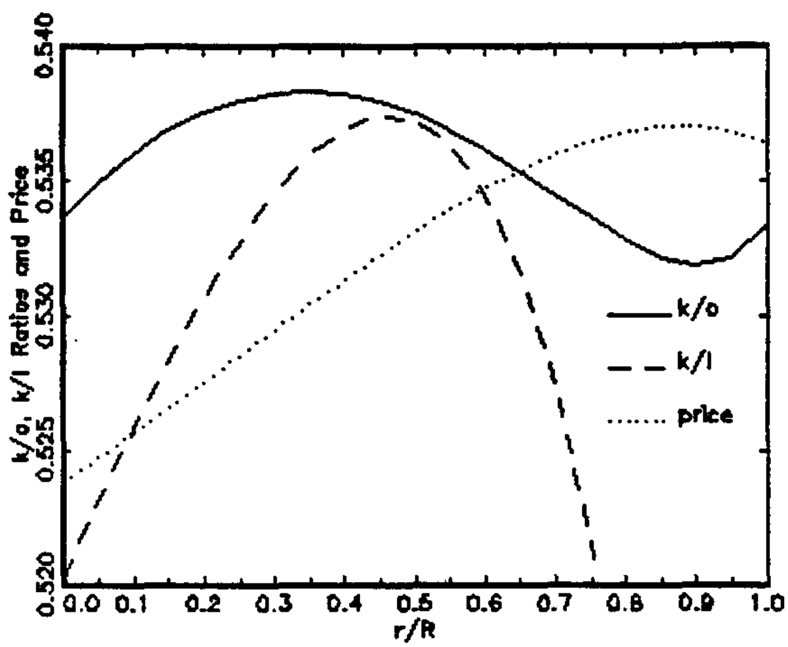
FIGURE 6.2.5

Sectoral Capital-Output, Capital-Labour Ratios and Price[17]
Bakery Products, 1972, Differential Profit Rate,
Sector 31 used as Profit Rate Denominator



17. In order to be able to graph the three curves together, the price vector of this industry has been multiplied by 2.

FIGURE 6.2.6
Interindustrial Capital-Output, Capital-Labour Ratios and Price[18] Leather and Leather Products, 1963, Uniform Profit Rate



On the other hand, for the Leather and Leather Products industry (18), the interindustrial capital-output ratio in 1963 under the uniform rate of profit scenario was a non-monotonic function of the rate of profit, also with two direction reversals increasing for initial values of r, decreasing at the intermediate range and then, at the higher end of the rate of profit range, increasing again. For their part its capital-labour ratio was a non-monotonic function of r with an

18. The capital-labour ratio and the price vector of this industry have been scaled by a factor of 2.00 and 2.25 respectively.

inverted "U" shape, while the own price was a non-monotonic function of the rate of profit. In this way from equation (13) we have that although the "own price effect" and the weighted "overall value of capital effect" were working in opposite directions, the latter was stronger and accounted for the first increasing segment of the capital-output. As the capital-labour ratio was approaching its maximum, its decrease in the rate of increase allowed the continuous increase in the "own price effect" to become the dominant force behind the movement of the capital-output ratio. At the higher end of the rate of profit, when the capital-labour was continuously decreasing, the decrease in the rate of increase of the price, as it was approaching its maximum, and its later decrease, accounted for a 'slow down' of the rate of decrease of the capital-labour ratio and its final increase. This is shown in Figure 6.2.6 below.

Changes in the Rate of Profit, the Capital-Output Ratios and Relative Prices: An Alternative View

Throughout our analysis we have stated that the rate of profit is an exogenous variable with respect to our system of equations, and that by setting the value of r we can determine prices and the wage rate in terms of a chosen numéraire. Moreover, the particular behaviour of these prices was placed at the centre of the explanation of the changes in the structure of the economy defined in terms of the capital-output and capital-labour ratios.

Every time we have calculated the system of prices, we have allowed the increase in the rate of profit to affect all the sectors at the same time, thus being unable to distinguish the effects of the increase in the price of any particular commodity whose individual rate of profit increased, so that the final movement may be the combined effect of changes in its own price and changes in the prices of all the other commodities for which the rate of profit also increased.

To "separate", as it were, the individual, industry by industry, effects of the change in the rate of profit, we rewrote our system of equations for the relative prices by postmultiplying matrix **A** by a diagonal matrix changing r only in the first industry. We then let that value of r increase from 0 to R[19] and observed the behaviour of ***all*** prices in terms of the value of the capital goods in that sector for which the rate of profit was changed (i.e. $\mathbf{pA}_1$). That is, we wrote:

$$p = a \cdot w + pA + pA \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \qquad \textbf{(18)}$$

and after rearranging this equation for **p** we divided the whole expression by $\mathbf{pA}_1$. Bear in mind that although we are allowing the rate of profit to be positive only in sector one, all prices will be affected. What is important to realize is that such changes must be the result of the increase in the rate of profit in that particular sector.

When the rate of profit had reached its maximum in sector 1, we then allowed the next sector to have a positive rate of profit, keeping the first sector's rate at the maximum and all the others equal to zero:

$$p = a \cdot w + pA + pA \begin{bmatrix} R & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \qquad \textbf{(19)}$$

Again, we increased the rate of profit in sector 2 from 0 to its maximum R, which is the same as for the first sector. We observed the new behaviour of all the prices, still measuring

19. Note that R could be replaced by any $0 \leq \bar{r} \leq R$.

prices in terms of the *first* sector's value of capital, so as to be able to make a straightforward comparison (i.e. keeping the same numéraire), as well as to allow us to analyse relative price changes in terms of changes in relative capital-output ratios, amongst other things. We repeated this procedure until each sector's rate of profit had been turned positive and allowed to reach the maximum value, R.

Since the point of the exercise was to see whether we could detect some simpler behaviour of the vector of prices as the rate of profit was increased in this step-by-step fashion, we chose as an example year 1967 because it was the year for which the most non-monotonic price vectors were found under the uniform rate of profit scenario. For the sake of simplicity, we aggregated the 1967 transactions matrix to 10 sectors -following a general but arbitrary classification between agriculture, manufacturing, commerce, services and government activities- and computed the new coefficient matrix. Then we rearranged the rows and columns so as to put first those sectors which we thought had more interindustrial linkages with the other sectors. (As an indicator of these linkages we used the rank order of the column sums of the **H** matrix). The order of the sectors was selected in order to maximize the effects of the accumulated increases in the rates of profit during the first couple of "iterations". In this way we expected to observe the most significant changes in relative prices in the first few steps of increasing one r_j at a time. We actually ran a first example with the original order (see the first two columns of Table 6.3.1 below) and observed in which iteration we found the most significant price changes. We then altered the order as shown in the third and fourth columns of that Table, using agggregated sectors (16 & 17) in Case A and (21 to 26) in Case B alternatively as first sectors (which also meant using their value of capital as numéraire in each case). In Table 6.3.1 we show the equivalence of the original 43 sectors with the aggregated 10 and the order in the new 10 x 10 matrix.

TABLE 6.3.1

Correspondence of the Original 43 Input-Output Sectors
with Aggregated 10 x 10 matrix

Number of Aggregated Sector	Original Input-Output Sector	Order in new 10 x 10 matrix A	B
1	1 - 4	4	4
2	5 - 11	6	6
3	12 - 15	8	8
4	16 - 17	1	2
5	18 - 20	3	3
6	21 - 26	2	1
7	27 - 28	9	9
8	29 - 30	8	8
9	31 - 40	5	5
10	41 - 43	10	10

We computed the new eigenvalue for the reduced 10 x 10 matrix and hence the new maximum rate of profit, which we would use to compute the range of the values of the rate of profit. With the computed prices we analysed, as we said before, the individual movement of the various price vectors. We can formalize this analysis, for the first Stage, in the following way:

$$p_1 = a_1 \cdot w + p(1+r_1) A_1$$
$$p_i = a_i \cdot w + pA_i \qquad (i = 2, 3, \ldots, n) \tag{20}$$

In terms of the value of capital in sector 1, all prices become:

Stage I: Only r_1 increases from 0 to R; all other r_i's=0.

$$\frac{p_1}{pA_1} = \frac{a_1 \cdot w}{pA_1} + (1+r_1)$$

$$\frac{p_i}{pA_1} = \frac{a_i \cdot w}{pA_1} + \frac{pA_i}{pA_1} \qquad (i \neq 1) \tag{21}$$

Then for

Stage II: When $r_1 = R$, $r_2 > 0$, all other r_i's $(i > 2)=0$.

$$\frac{p_1}{pA_1} = \frac{a_1 \cdot w}{pA_1} + (1+R)$$

$$\frac{p_2}{pA_1} = \frac{a_2 \cdot w}{pA_1} + (1+r_2)\frac{pA_2}{pA_1} \tag{22}$$

$$\frac{p_i}{pA_1} = \frac{a_i \cdot w}{pA_1} + \frac{pA_i}{pA_1} \qquad (i > 2)$$

and similarly for all other $i > 2$ when r_2 has reached its maximum value R.

It may be helpful, at this Stage, to consider an explicit analysis of the case in which i=3. Setting $\mathbf{pA}_1 = 1$, we could rewrite the two equations in (21) as:

Stage I:

Since $\mathbf{pA}_1 = 1$, we have:

$$p_1 = a_1 \cdot w + (1 + r)$$

$$[p_2, p_3] = [a_2, a_3]\, w + (a_1 \cdot w + (1+r)) \cdot [a_{12}, a_{13}] + [p_2, p_3] \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \tag{23}$$

$$= \left([a_2 , a_3]\, w + (a_1 \cdot w + (1+r)) [a_{12} , a_{13}] \right) \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1}$$

$$(a_1 w + (1+r))\, a_{11}$$

$$+ \left([a_2 , a_3] w + (a_1 w + (1+r)) [a_{12} , a_{13}] \right) \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1} \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix} = 1 \tag{24}$$

Rearranging and solving for w we get:

$$\frac{1-(1+r)\cdot\left(a_{11}+[a_{12}\ ,\ a_{13}]\left[I-\begin{bmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{bmatrix}\right]^{-1}\cdot\begin{bmatrix}a_{21}\\ a_{31}\end{bmatrix}\right)}{\dot{a}_1 a_{11}+\left([a_2\ ,\ a_3]+a_1[a_{12}\ ,\ a_{13}]\right)\cdot\left[I-\begin{bmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{bmatrix}\right]^{-1}\cdot\begin{bmatrix}a_{21}\\ a_{31}\end{bmatrix}} = w \tag{25}$$

It can be seen from equation (25) that the resulting wage-profit curve for this Stage I is a straight line. We must interpret this result as saying that in terms of the value of capital in sector 1, the "capital intensities" in both the other sectors remain invariant to changes in the distribution of income. Nevertheless since, in terms of the wage rate, all prices increase, but r changes only in sector 1, the price of that sector's commodity, in terms of the numéraire increases faster than all others, so that all other prices must decrease or at least, must not increase.

From the linearity of the wage-profit curve in this Stage I we can write:[20]

$$\dot{w} = -\frac{\frac{1}{v}}{R} = -\frac{\frac{1}{a_1}}{R+1} \tag{26}$$

From the derivative of p_1 with respect to r we get $\dot{p}_1 = a_1\ \dot{w} > 0$ given that $-\dot{w} < \frac{1}{a_1}$. Substituting this condition in equation (26) we have $\frac{1}{a_1} > \frac{1}{a_1(R+1)}$

20. See Steedman [1988] for the relationship between v^{-1} and a^{-1}, and thus R and R+1 in this equation.

which is of course true for all values of R $\neq 0$.[21] In the case of p_2, from its derivative with respect to r it can be seen a priori that $\dot{p}_2 < 0$ and also linear.[22]

More generally, we may write, for $\mathbf{pA}_1 = 1$ in Stage I:

$$[p_1, p_2, ..., p_n, w]\begin{bmatrix} a_{11} & 1 & -a_{12} & \cdots & -a_{1n} \\ a_{12} & 0 & (1-a_{22}) & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -a_1 & -a_2 & \cdots & -a_n \end{bmatrix} = [1, (1+r), 0, ..., 0] \quad (27)$$

Postmultiplying the right hand side vector by the inverse of the left hand side matrix (which we will denote **M**), we notice that, since the rate of profit does not appear inside that matrix, the resulting prices and wage rate are a ***linear*** function of the rate of profit. Moreover, only the first two rows of that inverse are relevant for the determination of prices, ***independently of the number of commodities considered***,[23] while it can be seen that the signs of the elements of the

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21. Note that for negative values of R equation (26) is rewritten as $\dot{w} = +\frac{\frac{1}{a_1}}{1-R}$ to account for the positive slope. As it can easily be verified, equation (26) would still be satisfied but with the above expression in the right hand side of the inequality.

22. Note that from (21): $\dot{p}_2 = a_2 \cdot \dot{w}$

23. This is because of the addition of more zeros in the right hand side vector.

second row will be positive for the first column and negative for all the others, thus making p_1 a linearly ***increasing*** function of the rate of profit r_1, while p_2, ..., p_n and w will all be linear but ***decreasing*** functions.

In terms of what happens in Stage II, we have:

Stage II:

$$p_1 = a_1 \cdot w + (1 + R)$$

$$p_2 = a_2 \cdot w + (1+r)\left[p_1 , p_2 , p_3\right] \cdot \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} \qquad (28)$$

$$p_3 = a_3 \cdot w + \left[p_1 , p_2 , p_3\right] \cdot \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

Following a similar procedure as for Stage I and keeping the value of capital in sector 1 as our numéraire, we obtain, for w:

$$\frac{1-(1+R)\cdot\left(a_{11}+\left[(1+r)a_{12},a_{13}\right]\left[I-\begin{bmatrix}(1+r)a_{22} & a_{23}\\(1+r)a_{32} & a_{33}\end{bmatrix}\right]^{-1}\begin{bmatrix}a_{21}\\a_{31}\end{bmatrix}\right)}{a_1a_{11}+\left(\left[a_2,a_3\right]+a_1\left[(1+r)a_{12},a_{13}\right]\right)\left[I-\begin{bmatrix}(1+r)a_{22} & a_{23}\\(1+r)a_{32} & a_{33}\end{bmatrix}\right]^{-1}\begin{bmatrix}a_{21}\\a_{31}\end{bmatrix}}=w \qquad \textbf{(29)}$$

Upon inspection it can be verified that the wage profit curve obtained for Stage II of our analysis is ***not*** a straight line, in general, so that now, in terms of the value of capital in sector 1, the capital intensities of the other sectors will indeed vary as the rate of profit is increased.

To further isolate the movement in relative prices in this Stage, from the observed movements in relative values of capital, we should re-express the system of prices in terms of the value of capital of sector two, i.e. using as numéraire $\mathbf{pA}_2=1$. Following the same procedure as that suggested in equation (28) we may now write:

$$[p_1, p_2, ..., p_n, w] \begin{bmatrix} a_{12} & 1-\rho a_{11} & 0 & \dots & -a_{1n} \\ a_{22} & -\rho a_{21} & 1 & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -a_1 & -a_2 & \dots & -a_n \end{bmatrix} = [1, 0, (1+r), ..., 0] \tag{30}$$

where $\rho = (1+R)$, and from which we can see, that it will be the first and third row of the inverse matrix which will be relevant in the determination of relative prices. Furthermore, as was the case in Stage I (when we used $\mathbf{pA}_1$ as numéraire), when we use as numéraire the value of capital of the commodity for which the rate of profit changes, prices become ***linear***, and from the elements in the third row of the inverse matrix, only p_2 is an increasing function of the rate of profit, while all the others prices ($p_i \neq p_2$) and w are decreasing functions of the rate of profit. It should be clear from the above equation that these results apply for the general case of "**n**" commodities, as in Stage I.

This alternative procedure has enabled us to separate not only the effect of the change in the rate of profit in each sector individually, but also, it has eliminated the efects of the change in the relative values of capital, as the rate of profit is increased, sector by sector. Of course, we can always go back to the uniform numéraire if at every Stage we multiply the resulting price vectors by the relative $\frac{pA_1}{pA_j}$ where j is the sector whose rate of profit is changed.

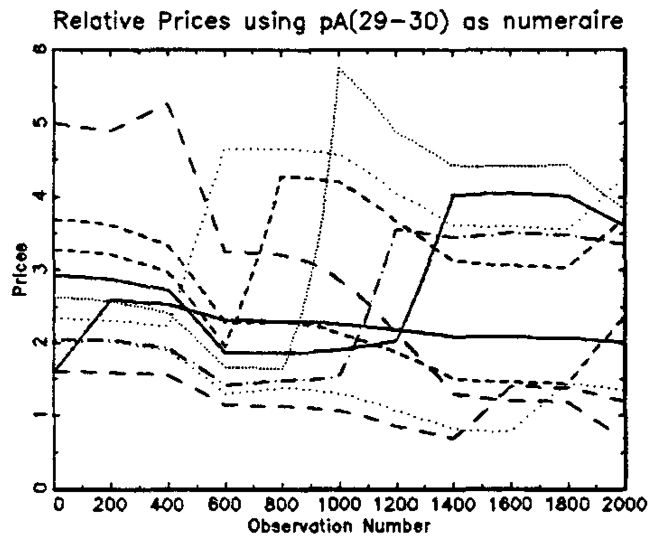
Note also that the prices used for the computation of these values of capital are the ones obtained by this process of increasing the rate of profit, sector by sector.

Moreover, we know from the earlier discussion in this paper that, the values of capital are affected by the particular behaviour of the overall vector of prices, weighted by the elements of the coefficient matrix. Thus, although we cannot, at this Stage make any specific statement about the movement of these relative values of capital, we can indeed say that we should expect these to change less than the respective relative prices.

Note that in the common analysis of changes in the system of relative prices, linear wage-profit curves meant that relative prices remained constant. In our case, what remains constant ***during Stage I of the analysis*** is the value of capital per worker, since we have taken the value of capital in the only sector for which the rate of profit changes to be equal to one, while in the other sectors r does not changes by construction. On the other hand relative prices do change, because although relative capital intensities remain the same, r changes only in sector 1, thus increasing its price faster than all others.

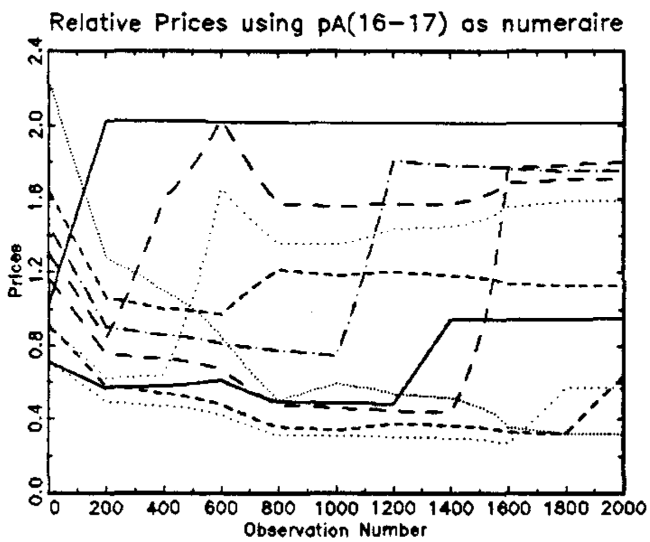
In Figures 6.3.1 and 6.3.2 overleaf we present the empirical results of this exercise for cases A and B described above, for the case of a uniform numéraire throughout the rate of profit changes in each sector. As we can see the behaviour of the relative prices has been greatly simplified in two major ways. First, prices have become more "linear" than in the common computation presented in the previous Chapter, becoming ***perfectly linear*** in Stage I. Secondly, it can be seen that the changes in each price are mostly due to changes in its own rate of profit, and that once this has been accounted for, some prices tend to become "nearly-constant", the "non-linearity" being brought to the system by the continuous change in the relative value of capital as the rate of profit changes sector by sector. That is to say that they are not much affected by increases in the rate of profit in other sectors.

FIGURE 6.3.1[24]



24. The increase in each sector's rate of profit was computed as R/200, so that in the horizontal axis we refer to "Observation Number" to mark the "Stage" of the analysis. In this way Stage I goes from observation 0 to 200, Stage II goes from observation 201 to 400, etc.

FIGURE 6.3.2



Concluding Remarks

In this paper we brought to the empirical level the implications of the theoretical developments of capital theory with a two fold purpose. We wanted first to see if the actual economic system was such that the results obtained at the theoretical level would still come out and, secondly, to show why we felt that traditional input-output theory fell short of its proposed analysis of structural change.

Jaime del Valle
May 1993

We saw how input-output theory identified structural change and technical progress and the way in which this identification hindered or prevented a thorough treatment of the concept of capital, along the lines shown by the results of the capital theory debate. We argued that since this framework begged the question of changes in the value of capital for unchanged methods of production, any attempt to extend this analysis in terms of the capital intensities or degrees of mechanization (as measured by the capital-output and capital-labour ratios) was, at the least, incomplete. Since we understood these variables to be at the centre of any analysis of structural change, we reformulated the analysis in terms of what we called the Sraffa-Pasinetti framework.

The reformulation of the analysis of structural change was carried out at two different levels: the interindustrial and the sectoral levels, where the basic difference between one and the other is the level of interrelationships accounted for in the measure of production inputs. The interindustrial analysis was based on the traditional input-output magnitudes of direct requirements while the sectoral analysis was based on Pasinetti's notion of vertical integration. In these terms we then formulated the expressions for the measurement of the capital-output and the capital-labour ratios, in a manner consistent with the basic results of capital theory. The most straightforward implication of this, which we saw in our formulation, is the introduction of the vector of prices and the distributive variables in the measures of capital. These alternative formulations of the capital-output and capital-labour ratio allowed us focus our analysis on the relationship between the behaviour of the system of prices and the particularity of the methods of production in shaping the behaviour of the capital-output and capital-labour ratios. In this way we saw how we could analyse the forces behind the movement of the capital-output ratios in terms of the movement of what we called the "overall value of capital effect", which reflects the changes in the capital-labour ratio, and the "own price effect" -which reflect the behaviour of that particular commodity price-, as the rate of profit is varied. At the same time the differentiation of the level of analysis allowed us to consider the capital intensity and degree of mechanization of the economic "system" even when considering individual industries.

Jaime del Valle
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After having analysed the implications of our framework we carried out our computations for the Puerto Rican economy for the 1963-1977 period. From our results we corroborated that the behaviour of the capital-output ratios, as expounded by Sraffian theory, were obtained at the empirical level, so that the capital-output and the capital-labour ratios did change with unchanged methods of production. The mere fact that these ratios changed makes conventional input-output theory "incomplete". Moreover, although most capital-output ratios changed monotonically with the rate of profit, some of these magnitudes changed in a non-monotonic way; we even observed two sectors in which this meant having ***two*** direction reversals. Another observation that we made from our results is that the selection of a differential rate of profit scenario, generally reduces the range of variation of the capital-output ratios throughout the whole range of the rate of profit, relative to the uniform rate of profit scenario. This we said could be interpreted as meaning that the allowance of differential profit rates makes different technical methods of production more "structurally similar". Finally in this respect, we showed, by means of two examples, the way in which the knowledge of the behaviour of the price vectors and the capital-labour ratios could be used to explain the way in which the web of interindustrial interrelationships ***must*** be affecting the capital-output ratio, and their relative importance in determing its movement.

Since the system of prices has been placed at the centre of the explanation of the behaviour of the capital-output and capital-labour ratios, in Section 6.3 we put forward an alternative analysis of the behaviour of the price system. In that section we tried to show that the behaviour of prices could be greatly simplified if we could "disect" or "separate" (i) the effect of the individual, industry by industry, changes n the rate of profit from the whole system of prices, and (ii) the effects of changes in the system of prices from the changes in the relative values of capital.

To do this we formalized a "Two-Stage" model in which, in the first stage, the rate of profit is increased only in industry "1"; all other industries' rate of profit being zero, and using as numéraire the value of capital in that industry. During "Stage II", it was the second industry's rate of profit which was allowed to increase, keeping constant that of industry "1", while all others industries' rate were still kept equal to zero. For this second stage we took, alternatively, the value of capital in the first industry, and later on, the value of its own capital.

It was shown by this exercise that when we change the rate of profit in only one industry at a time and use that industry's value of capital as numéraire, the wage-profit curves become ***perfectly linear*** and only the price of that industry whose rate of profit was allowed to change increased. All other prices were also linear but non-increasing.

On the other hand, when we went on to "Stage II" of the analysis, keeping as numéraire the value of capital in the first industry, it was shown that the wage-profit curves are ***not*** linear, so that relative prices will also change, although not necessarily in a monotonic way, given the fact that now relative capital intensities will also change as the rate of profit is varied. Nevertheless we could say that even in this second case, since the rate of profit is still changing in only one industry, change in the relative values of capital reflect only the behaviour caused by the changes system of prices brought about by changes in that industry's rate of profit. Since we know from the previous discusion that the behaviour of the value of capital is a result of the behaviour of the system of prices "weighted" by the relative importance of the elements of matrices **A** or **H**, we expect the change in the relative value of capital to be smaller than the change in the system of relative prices.

We made our analysis in a formal way, but by way of two examples we showed how in a first Stage the wage-profit curve was a straight line with prices changing in a linear way, with only the price of the sector for which the profit rate increased, increasing. At further Stages of the analysis, the behaviour of the prices was simpler if only because the price of the commodity for which the rate of profit changed increased faster than the others, while now other commodities prices' could also increase, because relative values of capital changed, given that the wage-profit curve was not a straight line.

When we changed the numéraire according to the sector for which the profit rate changed, (or according to the "Stage" of the analysis), we observed that the linearity of the changes in ***all*** prices was retained, as well as the uniquenes of the increasing nature of the particular price whose rate of profit was changed. All our results were generalized to an "n" commodities model.

Jaime del Valle
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